

## Agenda:

- 1) Bell Ringer: on easel
- 2) Lesson 4: Zero Exponents
- 3) Homework: Lesson 4 (1-5), ~~Lesson~~ 5 (1-4)  
*Lesson*
- 4) Exit Ticket (p.49)



## Lesson 4: Numbers Raised to the Zeroth Power

### Classwork

For any numbers  $x, y$ , and any positive integers  $m, n$ , the following holds

$$x^m \cdot x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

product rule → add exponents  
power rule → multiply exponents  
power rule (3)

Definition:

Exponent Rules

**Exercise 1**

List all possible cases of whole numbers  $m$  and  $n$  for identity (1). More precisely, when  $m > 0$  and  $n > 0$ , we already know that (1) is correct. What are the other possible cases of  $m$  and  $n$  for which (1) is yet to be verified?

$$1) 2^3 \cdot 2^5 = 2^8$$

$$2) (3^2)(3^6) = 3^8$$

Have not covered

$$3) 10^5 \cdot 10^{-5} = 10^0$$

$$4) 7^2 \cdot 7^{-6} = 7^{-4}$$

*Example 1:*  $4^0 = \underline{\hspace{1cm} 1 \hspace{1cm}}$ ,  
or 4 is used as a factor 0 times.

*Rule:* The value of an expression in which the exponent is 0 is 1.

$$\frac{2^3}{2^3} = 2^0 = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 1 = \frac{8}{8}$$

**Exercise 1**

Write the expanded form of 8,374 using exponential notation.

$$\begin{array}{r} 8000 + 300 + 70 + 4 \\ (8 \times 10^3) + (3 \times 10^2) + (7 \times 10^1) + (4 \times 10^0) \\ 8 \times 10^3 \qquad \qquad \qquad 3 \times 10^2 \qquad \qquad \qquad 7 \times 10 \qquad \qquad \qquad 4 \times 1 \end{array}$$

**Exercise 2**

Write the expanded form of 6,985,062 using exponential notation.

$$\begin{array}{r} 6,000,000 + 900,000 + 80,000 + 5,000 + 60 + 2 \\ (6 \times 10^6) + (9 \times 10^5) + (8 \times 10^4) + (5 \times 10^3) + (6 \times 10^2) + (2 \times 10^1) \\ 6 \times 10^6 \qquad \qquad \qquad 9 \times 10^5 \qquad \qquad \qquad 8 \times 10^4 \qquad \qquad \qquad 5 \times 10^3 \qquad \qquad \qquad 6 \times 10^2 \qquad \qquad \qquad 2 \times 1 \end{array}$$

**Homework**

Let  $x, y$  be numbers ( $x, y \neq 0$ ). Simplify each of the following expressions of numbers.

1.

$$\frac{y^{12}}{y^{12}} = y^0 = 1$$

2.

$$\frac{q^{15}}{q^{15}} \cdot \frac{1}{q^{15}} = \frac{q^{15}}{q^{15}} = q^0 = 1$$

3.

$$\frac{7^0}{1} \cdot \frac{(123456.789)^{10}}{1} = 1$$

4.

$$\frac{2^2}{1} \cdot \frac{1}{2^5} \cdot \frac{2^5}{1} \cdot \frac{1}{2^2} = \frac{2^2 \cdot 2^5}{2^5 \cdot 2^2} = \frac{2^2}{2^2} = 2^0 = 1$$

5.

$$\frac{x^{41}}{y^{15}} \cdot \frac{y^{15}}{x^{41}} = \frac{x^{41} \cdot y^{15}}{x^{41} \cdot y^{15}} = x^0 y^0 = 1(1) = 1$$

## Lesson 5: Negative Exponents and the Laws of Exponents

### Classwork

**Definition:** For any positive number  $x$  and for any positive integer  $n$ , we define  $x^{-n} = \frac{1}{x^n}$ .

Note that this definition of negative exponents says  $x^{-1}$  is just the reciprocal  $\frac{1}{x}$  of  $x$ . *flip*

As a consequence of the definition, for a positive  $x$  and all integers  $b$ , we get

$$x^{-b} = \frac{1}{x^b}$$

### Exercise 1

Verify the general statement  $x^{-b} = \frac{1}{x^b}$  for  $x = 3$  and  $b = 5$ .

$$3^{-5} = \frac{1}{3^5}$$

$$\frac{3}{1} \xrightarrow{\text{flip}} \frac{1}{3}$$

$$\frac{10}{1} \xrightarrow{\text{flip}} \frac{1}{10}$$

$$\frac{2}{3} \xrightarrow{\text{flip}} \frac{3}{2}$$

**Exercise 2**What is the value of  $(3 \times 10^{-2})$ ?

$$\frac{3}{1} \times \frac{1^2}{10^2} = \frac{3}{10^2} = \frac{3}{10 \cdot 10} = \frac{3}{100} = .03$$

why?  $\frac{10^{-2}}{10^5} = 10^{-3} = \frac{\cancel{10} \cdot \cancel{10}}{\cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10}} = \frac{1}{10^3}$

**Exercise 3**What is the value of  $(3 \times 10^{-5})$ ?

$$3 \times \frac{1}{10^5} = \frac{3}{10^5} = \frac{3}{100000} = .00003$$

$$\frac{10^2}{10^7} = 10^{-5} = \frac{\cancel{10} \cdot \cancel{10}}{\cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10}} = \frac{1}{10^5}$$

For Exercises 4–9, write an equivalent expression, in exponential notation, to the one given and simplify as much as possible.

**Exercise 4**

$$5^{-3} =$$

$$\frac{1}{5^3}$$

**Exercise 5**

$$\frac{1}{8^9} =$$

$$8^{-9}$$

**Exercise 6**

$$3 \cdot 2^{-4} =$$

$$3 \cdot \frac{1}{2^4}$$

**Exercise 7**

Let  $x$  be a nonzero number.

$$x^{-3} =$$

**Exercise 8**

Let  $x$  be a nonzero number.

$$\frac{1}{x^9} =$$

**Exercise 9**

Let  $x, y$  be two nonzero numbers.

$$xy^{-4} =$$

We accept that for positive numbers  $x, y$  and all integers  $a$  and  $b$ ,

$$x^a \cdot x^b = x^{a+b}$$

$$(x^b)^a = x^{ab}$$

$$(xy)^a = x^a y^a.$$

We claim

$$\frac{x^a}{x^b} = x^{a-b} \quad \text{for all integers } a, b.$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \quad \text{for any integer } a.$$

**Exercise 10**

$$\frac{19^2}{19^5} =$$

**Exercise 11**

$$\frac{17^{16}}{17^{-3}} =$$

**Exercise 12**

$$\left(\frac{7}{5}\right)^{-4} =$$

Homework

1. Compute:  $3^3 \times 3^2 \times 3^1 \times 3^0 \times 3^{-1} \times 3^{-2} = 3^{3+2+1+0+(-1)+(-2)} = 3^3 = 27$

Compute:  $5^2 \times 5^{10} \times 5^8 \times 5^0 \times 5^{-10} \times 5^{-8} = 5^{2+10+8+0+(-10)+(-8)} = 5^2 = 25$

Compute for a nonzero number  $a$ :  $a^{\cancel{3}} \times a^{\cancel{2}} \times a^{\cancel{1}} \times a^{\cancel{0}} \times a^{\cancel{-1}} \times a^{\cancel{-2}} \times a^0 = a^0 = 1$

Write equivalent expressions for the following problems.

2.  $(17.6^{-1})^8 = 17.6^{-8} = \frac{1}{17.6^8}$

3.  $(y^{-1})^n = y^{-n} = \frac{1}{y^n}$

4.  $\frac{2.8^{-5}}{2.8^7} = 2.8^{-5-7} = 2.8^{-12} = \frac{1}{2.8^{12}}$

